Introduction: the language of mathematics in science

The aim of this book is to enable teachers, publishers, awarding bodies and others to achieve a common understanding of important terms and techniques related to the use of mathematics in the science curriculum for pupils aged 11–16.

Background

This publication was produced as part of the ASE/Nuffield project *The Language of Mathematics in Science* in order to support teachers of 11–16 science in the use of mathematical ideas in the science curriculum. This is an area that has been a matter of interest and debate for many years. Some of the concerns have been about problems of consistency in terminology and approaches between the two subjects. The project built on the approach of a previous ASE/Nuffield publication, *The Language of Measurement*, and the aims are rather similar: to achieve greater clarity and agreement about the way the ideas and terminology are used in teaching and assessment.

Two publications have been produced during the project. This publication, *The Language of Mathematics in Science: A Guide for Teachers of 11–16 Science*, provides an overview of relevant ideas in secondary school mathematics and where they are used in science. It aims to clarify terminology and to indicate where there may be problems in student understanding. The publication includes explanations of key ideas and terminology in mathematics, along with a glossary of terms. A second publication, *The Language of Mathematics in Science: Teaching Approaches*, uses teachers' accounts to outline different ways that science and mathematics departments have worked together, and illustrates various teaching approaches with examples of how children respond to different learning activities.

About this publication

In developing this publication, the starting point was to identify relevant mathematical key words that should be included in the glossary. These were selected on the basis that the ideas form an important aspect of the current 11–16 science curriculum. Many familiar terms that pupils should know from elementary work in mathematics are not included (e.g. multiplication), although some terms are included that are currently not commonly used in 11–16 science but are potentially useful for science teachers to know about (e.g. box plot). The definitions of the selected key words are given in the Glossary for teachers at the end of this publication.

Definitions are useful for clarity, but only go so far. The major part of the publication consists of ten chapters, each explaining 'clusters' of these key words so that their use can be seen in context. The chapters and the associated key words are based around 'kinds of things we do in science'. For example, 'Collecting data' is concerned with terms such as 'quantity', 'value', 'unit' and 'variable'; 'Dealing with variability' is concerned with terms such as 'distribution', 'uncertainty', 'mean' and 'outlier'.

The clusters of key words are included in a panel at the start of each chapter, and a complete list of these can be seen in the Overview of chapters. Note that a number of key words appear in more than one chapter. Within the chapters, the key words are indicated in ***blue underlined*** text: each of these is hyperlinked to the relevant entry in the glossary. Each entry has hyperlinks back to the relevant sections in the chapters, so the glossary also acts as an index.

The aims of the publication are to:

- provide an overview of the mathematics relevant to science that may be studied by pupils at secondary school
- indicate the relevance of the ideas to the activities undertaken in secondary school science
- clarify the meaning of the terms used where there are common misunderstandings or where there are different meanings in different contexts
- indicate as appropriate where there may be student misconceptions and problems in understanding
- identify, where relevant, approaches taken in mathematics teaching that may influence what is done in science lessons.

Although there is some discussion of the details of mathematical techniques and procedures, this is not intended to be comprehensive, since further information can be found in relevant mathematics references. Instead, the focus is on an understanding of the underlying principles of the use of mathematics in school science. The intention is that the booklet will be a useful day-to-day reference that teachers can use to clarify ideas, as well as being used to inform the production of schemes of work and in promoting collaboration with the mathematics department.

Mathematics and science

Consistency between mathematics and science is clearly desirable wherever possible: it is unhelpful to have arbitrary differences in approaches and terminology between the subjects. This publication has been developed in collaboration with both science and mathematics educators in order to ensure that the usage of the language is correct and, as far as possible, consistent across subjects. However, mathematics and science are different disciplines, each with its own purposes, traditions and practices, and this leads to some differences in the way language is used.

In science, we are used to terms such as 'power', 'force', 'pressure' and so on having different meanings in everyday language, compared with the precise definitions used in science. There are fewer words in the language than meanings in the world to be expressed. It is not that science is correct and everyday language is wrong, but that words are used in different ways in different contexts. It is important for pupils to be able to recognise these differences when they move between contexts.

In a similar way, there are differences in the way that some terms are used in mathematics and science. One example is the term 'line', which has a more precise meaning in mathematics than the way it is often used in science. In mathematics, a line is, by definition, straight. In science, however, it is quite common to talk about 'straight lines' and 'curved lines'. Changing habitual ways of talking is hard: a good compromise in science might be to continue to refer to 'straight lines', but to talk of 'curves' rather than 'curved lines'. Another example is the use of the word 'histogram'. In mathematics, this refers to a display of a distribution of data in which the bars represent 'frequency density' for each class interval; in science, the bars of a histogram normally represent 'frequency'. Teachers and pupils need to be aware of such differences in usage between mathematics and science. Where these differences exist, they are indicated in the glossary and discussed in the relevant chapters.

In addition, some mathematical terms have multiple meanings in science; for example, 'range' may refer to the 'range of a measuring instrument', the 'range of an axis on a graph' or the 'range of a variable'. Other such examples include 'scale', 'coefficient' and 'variable'. Again, these differences in meanings are given in the glossary.

Collaboration between mathematics and science departments is clearly helpful in achieving a common understanding and in sequencing the introduction and use of mathematical ideas in an appropriate way. It is hoped that this publication will be useful in stimulating and supporting such discussions. However, it is beyond the scope of the publication to recommend details of what should be taught and when: this will depend on particular circumstances. Similarly, it is beyond the scope of the publication to give an indication of what range of knowledge or skills might be expected of pupils of different ages. While the current curricula in the UK have been taken into account in developing this publication, it has not been the intention to make any specific references to any particular programmes of study or assessment arrangements.

Thinking about purposes and using judgement

Although this book is designed to be used as a reference source, with sufficient cross-references that any section can be read independently, there is also a narrative that runs through it. Chapters 1 and 2 are concerned with the collection and processing of data, and Chapters 3 and 4 deal with the representation of data in tables, charts and graphs. Chapters 5 to 8 look at different kinds of relationships, from those where one variable is directly proportional to another to those where there is a good deal more variability in the data values. Finally, Chapter 9 focuses on the use of algebraic equations in science, and Chapter 10 looks at some of the areas of science that are also addressed in the mathematics curriculum.

One of the themes that runs throughout these chapters is the importance of thinking about *purposes*. What is the purpose of drawing this graph? What is the purpose of calculating this mean? It is not about the unthinking application of techniques but about considering what makes sense in different contexts. Knowing how to apply mathematical ideas in science is often a matter of using *judgement*.

There are no 'recipes' for how to design experiments, collect data and handle results; there are also no 'rules' for when and how to use mathematical techniques in science. This does not mean that nothing can be seen as 'correct' or 'incorrect'. It is important, for example, that pupils understand the meaning of 'significant figures', can distinguish this from 'decimal places' and can correctly apply the conventional rules of rounding. This is different from deciding what is an appropriate number of significant figures, which depends on context and requires judgement. These judgements are not uninformed – there are criteria that can be learned and that can support pupils' justifications for their decisions. It is for this reason that the publication puts an emphasis on understanding the nature of the mathematical ideas, and not just the techniques.

Understanding the nature of data

One table of data may look much like another, with numbers in rows and columns. Even though two tables may show superficial similarities, there may be fundamental differences between the nature of the data in each table. Such differences are important, since the ways that data can be analysed and how they can be represented visually depend on what kind of data they are. These choices may be difficult. Drawing a bar graph requires a certain level of competence, but deciding whether a bar chart is appropriate for a particular set of data is harder. Such questions have generated a good deal of discussion during the development of this publication. While much of this may be beyond many pupils, understanding the nature of different types of data is important in designing appropriate classroom activities.

There are a wide range of terms used in statistics to describe the nature of data. Chapter 1 provides a simplified account of the key ideas, with a focus on the distinctions that are helpful to make in science and on the questions that are useful to ask. It introduces the terms 'continuous', 'discrete' and 'categorical', which are revisited in Chapter 3 in considering how to represent data. There are many ways in which data can be organised and represented – in different kinds of tables, pie charts, bar charts, line graphs, scatter graphs and so on. The choices depend on the nature of the data and the kinds of questions about the data that are of interest. A particular advantage of a computer is that it allows different choices of display to be easily explored.

An important distinction is made in Chapter 3 between 'line graphs' and 'scatter graphs'. Although in one way they are similar, with the positions of data points determined by the scales on the horizontal and vertical axes, the nature of the data is fundamentally different. For example, experiments involving two continuous variables are particularly common in the physical sciences. Such experiments lead to 'line graph' type data with variability due to measurement uncertainty. On the other hand, surveys that collect data for which the variability is due to differences between individuals are more common in the biological sciences, and lead to 'scatter graph' type data. The guidance in the publication puts a good deal of emphasis on the importance of distinguishing between these two sources of variability in data. The differences are discussed in Chapter 6, with each of these being followed up in more detail in Chapters 7 and 8.

Interestingly, when such examples have been discussed by teachers, some of the comments have been about how teachers in the biological and physical sciences have different practices when dealing with data and constructing graphs. However, it is not so much that different sciences approach data in different ways; rather, different sciences often deal with different kinds of data, and different kinds of data are analysed in different ways.

Assessment

It is beyond the scope of this publication to deal with questions such as how mathematics in science could be assessed, or what pupils of different ages might be expected to know and do. However, the emphasis in the publication on thinking about the importance of purposes and judgement has implications in considering what is useful to be assessed. For example, if a pupil is asked to plot a set of data values on a graph with pre-drawn axes and scales, and to draw a line of best fit, how could the graph be judged?

- *The position of the data points on the graph*
  This requires the pupil to use values in a table and to read each of the scales correctly, putting a mark at each of the appropriate points on the graph. The positioning of the points is a matter of being 'correct' or 'incorrect'. Other such examples where there are 'right' and 'wrong' answers include rounding a value to a given number of significant figures, evaluating an expression and calculating the gradient of a straight line on a graph.
- *The choice of the line of best fit*
  While there may be some very obvious 'bad lines of fit', it is very unlikely that there would be a unique 'line of best fit': the choice requires judgement. Deciding on the position of a straight line, or whether to draw a straight line or a curve, or whether to include the origin, involves thinking about the meaning of the data and depends on the context. Other examples involving the use of judgement include deciding on an appropriate number of significant figures for a calculated value, choosing what kind of chart or graph to draw, and identifying which data values should be considered as outliers.

- *The symbols used for plotting the data points*
  This is a matter of convention, and there are various arguments for favouring one type of symbol over another. Pupils may meet different conventions in different published sources so they should be aware of this. However, while it is important for a publisher to have a consistent house style in a publication, it should not be a matter of importance for pupils to follow any particular convention. Other examples include the use of brackets or the solidus with units in the axis labels on a graph, or the use of words or letters for variables in a formula.

In assessing what pupils can do, it is important to distinguish between their competence in using particular techniques and the quality of their reasoning about how to use them. While there are no 'hard-and-fast rules' for how to draw a line of best fit or for the appropriate number of significant figures of a calculated value, this does not mean that 'anything goes'. This publication emphasises the kinds of considerations to be taken into account in order to make sensible judgements.

Process of development of this publication

The ASE began this project in summer 2014, after gaining funding from the Nuffield Foundation. The work of the project was informed by the advice of a steering group, and successive drafts of this publication were reviewed by a panel of science and mathematics educators with expertise in this area, and feedback obtained from a variety of groups of science teachers. During the project, discussions took place with representatives of the awarding organisations, who have been supportive of the approaches taken in this publication. An important consideration was that the recommendations in this publication should be realistic in practice, so the concluding stage of the process was a review of the draft guidance by a large panel of teachers before the production of the final publication. The time and effort spent by so many people in providing advice over the course of the project is much appreciated, and the quality of the publication has improved greatly as a result of it.

Further references on terminology and conventions

The following publications in particular were used to inform the use of terminology and conventions in this publication, and the definitions in the glossary.

1. *The Language of Measurement: Terminology Used in School Science Investigations* (2010). Hatfield: Association for Science Education. ISBN 978 0 86357 424 5.
   This publication contains a glossary, and selected terms and definitions from it are also included in the glossary of *The Language of Mathematics in Science*. Although there is common ground, duplication is avoided and the two publications should be seen as complementary.
2. *Signs, Symbols & Systematics: The ASE Companion to 16–19 Science* (2000). Hatfield: Association for Science Education. ISBN 978 0 86357 312 6.
   This publication is the definitive guide to a wide range of factual information related to 11–19 science education (and not just the 16–19 range suggested by the title). Of particular relevance to the areas covered by *The Language of Mathematics in Science* are sections on SI units, physical quantities, values of constants, and so on.
3. *Mathematics Glossary for Teachers in Key Stages 1 to 3* (2014). National Centre for Excellence in the Teaching of Mathematics.
   Many of the definitions in the glossary of *The Language of Mathematics in Science* are based on the NCETM glossary, as well as on the earlier QCA glossary for key stages 1 to 4, from which the NCETM version was adapted. It is available from the NCETM website (www.ncetm.org.uk/public/files/17308038/National+Curriculum+Glossary.pdf).